Resolution

Resolution is a rule of inference leading to a refutation theorem-proving technique for sentences in propositional logic and first-order logic. In other words, iteratively applying the resolution rule in a suitable way allows for telling whether a propositional formula is satisfiable and for proving that a first-order formula is unsatisfiable; this method may prove the satisfiability of a first-order satisfiable formula, but not always, as it is the case for all methods for first-order logic. Resolution was introduced by John Alan Robinson in 1965.

**Resolution in propositional logic**

The resolution rule in propositional logic is a single valid inference rule that produces a new clause implied by two clauses containing complementary literals. A literal is a propositional variable or the negation of a propositional variable. Two literals are said to be complements if one is the negation of the other (in the following, ai is taken to be the complement to bj). The resulting clause contains all the literals that do not have complements. Formally:



where

all as and bs are literals,
ai is the complement to bj, and
the dividing line stands for entails

The clause produced by the resolution rule is called the resolvent of the two input clauses.

When the two clauses contain more than one pair of complementary literals, the resolution rule can be applied (independently) for each such pair. However, only the pair of literals that are resolved upon can be removed: all other pair of literals remain in the resolvent clause.

**A resolution technique**

When coupled with a complete search algorithm, the resolution rule yields a sound and complete algorithm for deciding the satisfiability of a propositional formula, and, by extension, the validity of a sentence under a set of axioms.

This resolution technique uses proof by contradiction and is based on the fact that any sentence in propositional logic can be transformed into an equivalent sentence in conjunctive normal form. The steps are as follows:

1).All sentences in the knowledge base and the negation of the sentence to be proved (the conjecture) are conjunctively connected.

2).The resulting sentence is transformed into a conjunctive normal form with the conjuncts viewed as elements in a set, S, of clauses.

For example



would give rise to a set



3).The resolution rule is applied to all possible pairs of clauses that contain complementary literals. After each application of the resolution rule, the resulting sentence is simplified by removing repeated literals. If the sentence contains complementary literals, it is discarded (as a tautology). If not, and if it is not yet present in the clause set S, it is added to S, and is considered for further resolution inferences.

4).If after applying a resolution rule the empty clause is derived, the complete formula is unsatisfiable (or contradictory), and hence it can be concluded that the initial conjecture follows from the axioms.

5).If, on the other hand, the empty clause cannot be derived, and the resolution rule cannot be applied to derive any more new clauses, the conjecture is not a theorem of the original knowledge base.

**Example**



In English: if a or b is true, and a is false or c is true, then either b or c is true.

If a is true, then for the second premise to hold, c must be true. If a is false, then for the first premise to hold, b must be true.

So regardless of a, if both premises hold, then b or c is true.